ISSN: 2454-9940

# INTERNATIONAL JOURNAL OF APPLIED SCIENCE ENGINEERING AND MANAGEMENT

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www.ijasem.org

INTERNATIONAL JOURNAL OF APPLIED SCIENCE ENGINEERING AND MANAGEMENT

ISSN2454-9940www.ijsem.org

Vol 6, Issuse.4 Oct 2017

# Considerations for the Generalized Finite Difference Method in Dynamic Analysis

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**ABSTRACT**

***In this study, the generalized finite difference technique (GFDM) is used to perform a dynamic analysis of beams and plates. For beams and plates, we provide the stability criteria for an entirely explicit method. Beam and plate point cloud irregularity measures are provided. Results from solving problems involving beam and plate vibrations demonstrate the reliability of the method for clouds of nodes with irregular shapes.***

## INTRODUCTION

From the traditional finite difference approach came the generalized finite difference method (GFDM) (FDM). It doesn't matter whether the point cloud you're working with is uniform or very irregular; GFDM may be used on it [1]. The goal is to employ a method called moving least squares approximation to derive explicit difference formulas that can be plugged into the partial differential equations [2]. Benito, Urea, and Gavete have made several promising contributions [3-8] to the refinement of this technique. The GFDM is used to solve hyperbolic and parabolic equations, as shown in [9]. In this study, we provide an explicit approach [10-13] for employing the GFDM to solve dynamic analytic issues involving beams and plates. Specifically, the paper follows this structure. The first part is an introduction. Section 2.1 details the explicit GFDM scheme for beams, and Section 2.2 details the explicit GFDM strategy for plates, both of which are described in Section 2 of this work.

7.

There are two types of truncation errors that are studied in this paper: beam truncation errors in Section 3.1.1 and plate truncation errors in Section 3.1.2. Section 3 focuses on the convergence, consistency, and von Neumann stability. In Section 3.2.1, we examine von Neumann stability for beams, and in Section 3.2.2, we do the same for plates. In Section 4, we look at how the consistency of a cluster of nodes is related to its erratic nature. The index of irregularity of a cloud of nodes is defined for beams in Section 4.1, and for plates in Section 4.2. Some GFDM for solvin applications are discussed in Section 5. Included are difficulties in doing a dynamic study of beams. In Section 6, we look at how the GFDM has been used to address issues in the field of dynamic analysis of plates. Finally, some findings are presented in Section

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## Explicit generalized finite difference schemes

Frequency of beam vibrations in beams are the first kind of example we'll look at. So, let's take into account the situation described by the following partial differential equation (pde)

$$\frac{\partial^2 U(x,t)}{\partial t^2} + C_1^2 \frac{\partial^4 U(x,t)}{\partial x^4} = F_1(x,t) \quad x \in (0,L), t > 0 \tag{1}$$

Starting and case-specific boundary conditions for a beam of length L

$$U(x,0) = 0; \quad \left.\frac{\partial U(x,t)}{\partial t}\right|_{(x,0)} = F_2(x) \tag{2}$$

Where F1 and F2 are constants, while the material and shape of the beam determine C1.

For the values of partial derivatives in the space variable, we use the explicit difference equations. The goal is to find closed-form linear equations that may be used to approximate partial derivatives at certain places in the domain. Before doing anything further, the domain is seeded with an uneven grid or cloud of points. After establishing a composition's central node and the N points (henceforth nodes) around it, the star may be used to refer to the formed node group in respect to the central node. All of the domain's vertices have been given a star in the order [3, 2, 4, and 1]. If the value of the function at the central node (U0) of the star, at coordinate x0, is approximated by the fourth-order approximation u0, and the values of the function at the other nodes, at coordinates xj with j = 1,..., N, are approximated by the fourth-order approximations uj, then the Taylor series expansion states that.

$$U_j = U_0 + h_j \frac{\partial U_0}{\partial x} + \frac{h_j^2}{2}\frac{\partial^2 U_0}{\partial x^2} + \frac{h_j^3}{6}\frac{\partial^3 U_0}{\partial x^3} + \frac{h_j^4}{24}\frac{\partial^4 U_0}{\partial x^4} + \cdots \tag{3}$$

where $h_j = x_j - x_0$.

Function B4(u) may be defined as in if the terms above the fourth order in Eq. (3) are disregarded.

$$B_4(u) = \sum_{j=1}^{N} \left[ \left( u_0 - u_j + h_j \frac{\partial u_0}{\partial x} + \frac{h_j^2}{2}\frac{\partial^2 u_0}{\partial x^2} + \frac{h_j^3}{6}\frac{\partial^3 u_0}{\partial x^3} + \frac{h_j^4}{24}\frac{\partial^4 u_0}{\partial x^4} \right) w(h_j) \right]^2 \tag{4}$$

And the denominator weighting function is represented by w(hj). The system of linear equations is produced if the norm (4) is minimized with regard to the partial derivatives.

$$A_4 D_{u_4} = b_4 \tag{5}$$

where

$$A_4 = \begin{pmatrix} \sum_{j=1}^{N} h_j^2 w^2 & \sum_{j=1}^{N} \frac{h_j^3}{2} w^2 & \sum_{j=1}^{N} \frac{h_j^4}{6} w^2 & \sum_{j=1}^{N} \frac{h_j^5}{24} w^2 \\ & \sum_{j=1}^{N} \frac{h_j^4}{4} w^2 & \sum_{j=1}^{N} \frac{h_j^5}{12} w^2 & \sum_{j=1}^{N} \frac{h_j^6}{48} w^2 \\ & & \sum_{j=1}^{N} \frac{h_j^6}{36} w^2 & \sum_{j=1}^{N} \frac{h_j^7}{144} w^2 \\ & SYM & & \sum_{j=1}^{N} \frac{h_j^8}{576} w^2 \end{pmatrix} \tag{6}$$

and

$$D_{u_4} = \left\{ \frac{\partial u_0}{\partial x} \quad \frac{\partial^2 u_0}{\partial x^2} \quad \frac{\partial^3 u_0}{\partial x^3} \quad \frac{\partial^4 u_0}{\partial x^4} \right\}^T \tag{7}$$

$$b_4 = \begin{Bmatrix} \sum_{j=1}^{N} (-u_0 + u_j) h_j w^2 \\ \sum_{j=1}^{N} (-u_0 + u_j) \frac{h_j^2}{2} w^2 \\ \sum_{j=1}^{N} (-u_0 + u_j) \frac{h_j^3}{6} w^2 \\ \sum_{j=1}^{N} (-u_0 + u_j) \frac{h_j^4}{24} w^2 \end{Bmatrix} \tag{8}$$

By using the same procedure as in [3-5, 9], the explicit difference formulas for the fifth system are produced. When the partial derivatives' values are given in explicit form, the star equation is achieved.

$$\left.\frac{\partial^4 U(x,t)}{\partial x^4}\right|_{(x_0,n\Delta t)} = \eta_0 u_0 + \sum_{j=1}^{N} \eta_j u_j \tag{9}$$

with

$$\eta_0 + \sum_{j=1}^{N} \eta_j = 0. \tag{10}$$

The time-dependent portion of Eq. (1) will be calculated using an explicit formula. The Cauchy starting value issue may be solved by using this explicit formula. This technique requires only a single grid point at the maximum time scale. An approximation to the second derivative with regard to time

$$\left.\frac{\partial^2 U}{\partial t^2}\right|_{(x_0,n\Delta t)} = \frac{u_0^{n+1} - 2u_0^n + u_0^{n-1}}{(\Delta t)^2}. \tag{11}$$

When Equations (9) and (11) are inserted into Eq. (1), the following recursive connection is obtained:

$$u_0^{n+1} = 2u_0^n - u_0^{n-1} - C_1^2(\Delta t)^2\left[\eta_0 u_0^n + \sum_{j=1}^{N} \eta_j u_j^n\right] + F_1(x_0, n\Delta t). \tag{12}$$

The central difference formula is a method of approximating the first derivative with regard to time.

$$\left.\frac{\partial U}{\partial t}\right|_{(x_0,0)} = \frac{u_0^1 - u_0^{-1}}{2\Delta t} = F_2(x_0) \Rightarrow u_0^{-1} = u_0^1 - 2\Delta t F_2(x_0). \tag{13}$$

The following equation is found by substituting Eq. (13) into Eq. (12) while also considering beginning conditions (2):

$$u_0^1 = \Delta t F_2(x_0) + \frac{F_1(x_0, 0)}{2}. \tag{14}$$

The value of the function at the centre node of the star at time n = 1 is related to the values F1(x0, 0) and the starting conditions F2 by Eq. (14). (x0).

2.2. Vibrations of plates

Vibrations of plates are the second scenario we analyze. Let's think about the situation in terms of

$$\frac{\partial^2 U(x,y,t)}{\partial t^2} + C_2^2\left[\frac{\partial^4 U(x,y,t)}{\partial x^4} + 2\frac{\partial^4 U(x,y,t)}{\partial x^2 \partial y^2} + \frac{\partial^4 U(x,y,t)}{\partial y^4}\right] = G_1(x,y,t)$$
$$(x,y) \in (0,L) \times (0,L), t > 0 \tag{15}$$

Including starting and plate boundary conditions in the range [0, L] [0, L].

$$U(x,y,0) = 0; \quad \left.\frac{\partial U(x,y,t)}{\partial t}\right|_{(x,y,0)} = G_2(x,y) \tag{16}$$

Where G1 and G2 are two smooth functions of known value and C2 is a constant that varies with plate material and shape. Fourth-order approximations of the value of the function at the centre node (U0) of the star, with coordinates (x0, y0), u0, and uj, are employed in the same manner as in the preceding subsection.

Then, using the Taylor series expansion, we can determine the value of the function at the remaining nodes, or coordinates (xj, yj) for j = 1,..., N.

$$U_j = U_0 + h_j\frac{\partial U_0}{\partial x} + k_j\frac{\partial U_0}{\partial y} + \frac{h_j^2}{2}\frac{\partial^2 U_0}{\partial x^2} + \frac{k_j^2}{2}\frac{\partial^2 U_0}{\partial y^2} + h_j k_j\frac{\partial^2 U_0}{\partial x \partial y}$$
$$+ \frac{h_j^3}{6}\frac{\partial^3 U_0}{\partial x^3} + \frac{k_j^3}{6}\frac{\partial^3 U_0}{\partial y^3} + \frac{h_j^2 k_j}{2}\frac{\partial^3 U_0}{\partial x^2 \partial y} + \frac{h_j k_j^2}{2}\frac{\partial^3 U_0}{\partial x \partial y^2} + \frac{h_j^4}{24}\frac{\partial^4 U_0}{\partial x^4} + \frac{k_j^4}{24}\frac{\partial^4 U_0}{\partial y^4}$$
$$+ \frac{h_j^3 k_j}{6}\frac{\partial^4 U_0}{\partial x^3 \partial y} + \frac{h_j^2 k_j^2}{4}\frac{\partial^4 U_0}{\partial x^2 \partial y^2} + \frac{h_j k_j^3}{6}\frac{\partial^4 U_0}{\partial x \partial y^3} + \cdots \tag{17}$$

where $h_j = x_j - x_0; k_j = y_j - y_0$.

Ignore the terms above the fourth order in Eq. (17). The function may then be defined.

$$B_{14}(u) = \sum_{j=1}^{N}\left[\left(u_0 - u_j + h_j\frac{\partial u_0}{\partial x} + k_j\frac{\partial u_0}{\partial y} + \frac{h_j^2}{2}\frac{\partial^2 u_0}{\partial x^2} + \frac{k_j^2}{2}\frac{\partial^2 u_0}{\partial y^2} + h_j k_j\frac{\partial^2 u_0}{\partial x \partial y}\right.\right.$$
$$+ \frac{h_j^3}{6}\frac{\partial^3 u_0}{\partial x^3} + \frac{k_j^3}{6}\frac{\partial^3 u_0}{\partial y^3} + \frac{h_j^2 k_j}{2}\frac{\partial^3 u_0}{\partial x^2 \partial y} + \frac{h_j k_j^2}{2}\frac{\partial^3 u_0}{\partial x \partial y^2} + \frac{h_j^4}{24}\frac{\partial^4 u_0}{\partial x^4} + \frac{k_j^4}{24}\frac{\partial^4 u_0}{\partial y^4}$$
$$\left.\left.+ \frac{h_j^3 k_j}{6}\frac{\partial^4 u_0}{\partial x^3 \partial y} + \frac{h_j^2 k_j^2}{4}\frac{\partial^4 u_0}{\partial x^2 \partial y^2} + \frac{h_j k_j^3}{6}\frac{\partial^4 u_0}{\partial x \partial y^3}\right) w(h_j, k_j)\right]^2 \tag{18}$$

For some w (hj, kJ) where w (hj, kJ) is the denominator weighting function.

The system of linear equations is produced if the norm (18) is minimized with regard to the partial derivatives.

$$A_{14} D_{u_{14}} = b_{14}$$

where

$$A_{14} = \begin{pmatrix} \sum_{j=1}^{N} h_j^2 w^2 & \sum_{j=1}^{N} h_j k_j w^2 & \sum_{j=1}^{N} \frac{h_j^3}{2} w^2 & \sum_{j=1}^{N} \frac{h_j k_j^2}{2} w^2 & \sum_{j=1}^{N} h_j^2 k_j w^2 & \sum_{j=1}^{N} \frac{h_j^4}{6} w^2 & \cdots & \sum_{j=1}^{N} \frac{h_j^3 k_j^2}{4} w^2 \\ & \sum_{j=1}^{N} k_j^2 w^2 & \sum_{j=1}^{N} \frac{h_j^2 k_j}{2} w^2 & \sum_{j=1}^{N} \frac{k_j^3}{2} w^2 & \sum_{j=1}^{N} h_j k_j^2 w^2 & \sum_{j=1}^{N} \frac{h_j^3 k_j}{6} w^2 & \cdots & \sum_{j=1}^{N} \frac{h_j^2 k_j^3}{4} w^2 \\ & & \sum_{j=1}^{N} \frac{h_j^4}{4} w^2 & \sum_{j=1}^{N} \frac{h_j^2 k_j^2}{4} w^2 & \sum_{j=1}^{N} \frac{h_j^3 k_j}{2} w^2 & \sum_{j=1}^{N} \frac{h_j^5}{12} w^2 & \cdots & \sum_{j=1}^{N} \frac{h_j^4 k_j^2}{8} w^2 \\ & & & \sum_{j=1}^{N} \frac{k_j^4}{4} w^2 & \sum_{j=1}^{N} \frac{h_j k_j^3}{2} w^2 & \sum_{j=1}^{N} \frac{h_j^3 k_j^2}{12} w^2 & \cdots & \sum_{j=1}^{N} \frac{h_j^2 k_j^4}{8} w^2 \\ & & & & \sum_{j=1}^{N} h_j^2 k_j^2 w^2 & \sum_{j=1}^{N} \frac{h_j^4 k_j}{6} w^2 & \cdots & \sum_{j=1}^{N} \frac{h_j^3 k_j^3}{4} w^2 \\ & & & & & \sum_{j=1}^{N} \frac{h_j^6}{36} w^2 & \cdots & \sum_{j=1}^{N} \frac{h_j^5 k_j^2}{24} w^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ & S & & Y & & M & \cdots & \sum_{j=1}^{N} \frac{h_j^4 k_j^4}{16} w^2 \end{pmatrix} \quad (20)$$

and

$$D_{u_{14}} = \left\{ \frac{\partial u_0}{\partial x} \quad \frac{\partial u_0}{\partial y} \quad \frac{\partial^2 u_0}{\partial x^2} \quad \frac{\partial^2 u_0}{\partial y^2} \quad \frac{\partial^2 u_0}{\partial x \partial y} \quad \frac{\partial^3 u_0}{\partial x^3} \quad \cdots \quad \frac{\partial^4 u_0}{\partial x \partial y^3} \quad \frac{\partial^4 u_0}{\partial x^2 \partial y^2} \right\}^T \quad (21)$$

$$b_{14} = \left\{ \sum_{j=1}^{N} (-u_0 + u_j) h_j w^2 \quad \cdots \quad \sum_{j=1}^{N} (-u_0 + u_j) \frac{h_j^3}{6} w^2 \quad \cdots \quad \sum_{j=1}^{N} (-u_0 + u_j) \frac{h_j^2 k_j^2}{4} w^2 \right\}^T \quad (22)$$

Moreover, the explicit difference formulas are produced by solving the system. When the partial derivatives' values are given in explicit form, the star equation is achieved.

$$\left[ \frac{\partial^4 U(x,y,t)}{\partial x^4} + 2\frac{\partial^4 U(x,y,t)}{\partial x^2 \partial y^2} + \frac{\partial^4 U(x,y,t)}{\partial y^4} \right]_{(x_0, y_0, n\Delta t)} = \mu_0 u_0 + \sum_{j=1}^{N} \mu_j u_j \quad (23)$$

with

$$\mu_0 + \sum_{j=1}^{N} \mu_j = 0. \quad (24)$$

An approximation to the second derivative with regard to time

$$\left. \frac{\partial^2 U}{\partial t^2} \right|_{(x_0, y_0, n\Delta t)} = \frac{u_0^{n+1} - 2u_0^n + u_0^{n-1}}{(\Delta t)^2}. \quad (25)$$

When replaced in Eq. (15), the recursive connection shown below is obtained:

$$u_0^{n+1} = 2u_0^n - u_0^{n-1} + G_2^2 (\Delta t)^2 \left[ \mu_0 u_0^n + \sum_{j=1}^{N} \mu_j u_j^n \right] + G_1(x_0, y_0, n\Delta t). \quad (26)$$

The central difference formula approaches the first time derivative.

$$\left. \frac{\partial U}{\partial t} \right|_{x_0, y_0, 0} = \frac{u_0^1 - u_0^{-1}}{2\Delta t} = G_2(x_0, y_0) \Rightarrow u_0^{-1} = u_0^1 - 2\Delta t G_2(x_0, y_0). \quad (27)$$

The following equation is produced when Eq. (27) is substituted into Eq. (26) while starting conditions (16) are considered:

$$u_0^1 = \Delta t G_2(x_0, y_0) + \frac{G_1(x_0, y_0, 0)}{2}. \quad (28)$$

The expressions (12) and (26) show a connection between the value of the function at the central node of the star at time step n + 1 and the values of the functions at the nodes of the star at time step n.

By solving the systems of Eqs. (5) And (19) and plugging in the partial derivatives, the explicit generalized finite difference schemes may be produced. All of the partial derivatives in the PDE must be accounted for in the Taylor series expansion. This is because the GFD scheme is derived in a manner unique to the partial differential equation being solved. The Taylor series expansion (Eqs. (3) and (17)) relies on partial derivatives; hence this also depends on the weighting function (see [3, 4, and 9]).

**Convergence**

Stability is sufficient for convergence under Lax's equivalence theorem, provided that the consistency criterion is met. Here, we look at the truncation error of Eqs. (12) (For beams) and (26) (for plates), then we go on to stability**.**

**Truncation error and consistency**

Beams

It is well knowledge that the following expression describes the truncation errors for the second order time derivative (TEt).

$$\frac{\partial^2 U(x,t)}{\partial t^2} = \frac{u_0^{t+\Delta t} - 2u_0^t + u_0^{t-\Delta t}}{(\Delta t)^2} - \frac{(\Delta t)^2}{12} \frac{\partial^4 U(x,t_1)}{\partial t^4} + \theta((\Delta t)^4), \quad t < t_1 < t + \Delta t \quad (29)$$

$$(TE_t) = -\frac{(\Delta t)^2}{12} \frac{\partial^4 U(\mathbf{x}, t_1)}{\partial t^4} + \theta((\Delta t)^4), \quad t < t_1 < t + \Delta t. \quad (30)$$

Higher order functions B 4 [u] are derived by using the Taylor series expansion that includes higher order derivatives to determine the truncation error for space derivatives. Similar to the formulations of B 4 [u] provided by Eq. (4), but with higher-order derivatives included.

$$B_4(u) = \sum_{j=1}^{N} \left[ \left( u_0 - u_j + h_j \frac{\partial u_0}{\partial x} + \frac{1}{2!} \frac{h_j^2 \partial^2 u_0}{\partial x^2} + \frac{1}{3!} \frac{h_j^3 \partial^3 u_0}{\partial x^3} + \frac{1}{4!} \frac{h_j^4 \partial^4 u_0}{\partial x^4} + \frac{1}{5!} \frac{h_j^5 \partial^5 u_0}{\partial x^5} + \frac{1}{6!} \frac{h_j^6 \partial^6 u_0}{\partial x^6} + \cdots \right) w(h_j) \right]^2. \quad (31)$$

The following systems of linear equations are formed if the new norms B 4 [u] are minimized with regard to the partial derivatives up to the fourth order:

$$A_4 D_{u_4} = b_4^* \tag{32}$$

Where A4, Du4, and b4 were determined using Eqs. (6) And (7), and b 4 is defined as

$$b_4^* = \left( \sum_{j=1}^{N} \Xi h_j \;\; \sum_{j=1}^{N} \Xi \frac{h_j^2}{2!} \;\; \sum_{j=1}^{N} \Xi \frac{h_j^3}{3!} \;\; \sum_{j=1}^{N} \Xi \frac{h_j^4}{4!} \right)^T \tag{33}$$

where

$$\Xi = \left[ -u_0 + u_j - \frac{h_j^5}{5!}\frac{\partial^5 u_0}{\partial x^5} - \frac{h_j^6}{6!}\frac{\partial^6 u_0}{\partial x^6} - \cdots \right] w(h_j)^2 \tag{34}$$

with $N \geq 4$.

It is possible to divide the value of b4 in half as shown below:

$$b_4^* = b_4 + b_4^{**}$$

For beams, the GFDM uses the approximation expressed by A1 4 b4 in Eq. (36); the truncation errors for spatial derivatives are thus given by

$$TE_{x_4} = A_4^{-1} b_4^{**}.$$

Thus

$$TE_{x_4} = -\frac{1}{C_1^2} A_4^{-1} \times \left( \sum_{j=1}^{N} \Upsilon h_j \;\; \sum_{j=1}^{N} \Upsilon \frac{h_j^2}{2!} \;\; \sum_{j=1}^{N} \Upsilon \frac{h_j^3}{3!} \;\; \sum_{j=1}^{N} \Upsilon \frac{h_j^4}{4!} \right)^T \tag{38}$$

where

$$\Upsilon = -\left[ \frac{h_j^5}{5!}\frac{\partial^5 U_0}{\partial x^5} + \frac{h_j^6}{6!}\frac{\partial^6 U_0}{\partial x^6} + \cdots \right] w(h_j)^2 \tag{39}$$

and operating

$$TE_{x_4} = \frac{1}{C_1^2} \left[ \sum_{j=1}^{N} \psi_{1,j} \frac{\partial^5 U}{\partial x^5} + \psi_{2,j} \frac{\partial^6 U}{\partial x^6} + \cdots \right] + \Theta(h_j) \tag{40}$$

Where 1,j(he) and 2,j(hj) are rational homogeneous functions of order one and two, respectively, and (hj) is a sequence of functions of order three or higher. In the case of beams, the truncation error for spatial derivatives is given by Expression (40). Remember that the sum of the truncation errors (TTE) is

$$TTE = TE_t + TE_{x_4} \tag{41}$$

Thus (30) and (41) may be used to get TEt and TEx4, respectively.

Since Eq.41

$$\lim_{(\Delta t, h_j) \to (0,0)} TTE \to 0. \tag{42}$$

Then, the approximation's consistency is shown by the truncation error condition in Eq. (42).

**Plates**

It is well knowledge that the truncation errors for the second order time derivative (TEt) are as follows.

$$(TE_t) = -\frac{(\Delta t)^2}{12}\frac{\partial^4 U(x, y, t_1)}{\partial t^4} + \Theta((\Delta t)^4), \quad t < t_1 < t + \Delta t. \tag{43}$$

Higher order functions B 14[u] are created by using the Taylor series expansion that includes higher order derivatives to calculate the truncation error for space derivatives. Similar to the formula in Eq. (18), but now including higher order derivatives, are the expressions B 14[u].

$$\begin{aligned} B_{14}^*(u) = \sum_{j=1}^{N} \Bigg[ \Bigg( & u_0 - u_j + h_j \frac{\partial u_0}{\partial x} + k_j \frac{\partial u_0}{\partial y} + \frac{1}{2}\left( h_j \frac{\partial u_0}{\partial x} + k_j \frac{\partial u_0}{\partial y} \right)^{(2)} \\ & + \frac{1}{3!}\left( h_j \frac{\partial u_0}{\partial x} + k_j \frac{\partial u_0}{\partial y} \right)^{(3)} + \frac{1}{4!}\left( h_j \frac{\partial u_0}{\partial x} + k_j \frac{\partial u_0}{\partial y} \right)^{(4)} \\ & + \frac{1}{5!}\left( h_j \frac{\partial u_0}{\partial x} + k_j \frac{\partial u_0}{\partial y} \right)^{(5)} + \frac{1}{6!}\left( h_j \frac{\partial u_0}{\partial x} + k_j \frac{\partial u_0}{\partial y} \right)^{(6)} + \cdots \Bigg) w(h_j, k_j) \Bigg]^2 . \end{aligned} \tag{44}$$

The following sets of linear equations are produced if the new norm B 14[u] is minimized with regard to the partial derivatives up to the fourth order:

$$A_{14} D_{u_{14}} = b_{14}^*$$

Where A14 and Du14 have been determined using Eqs. (20) And (21), respectively, and b 14 is defined as

$$b_{14}^* = \left( \sum_{j=1}^{N} \Xi h_j \;\; \sum_{j=1}^{N} \Xi k_j \;\; \sum_{j=1}^{N} \Xi \frac{h_j^2}{2!} \cdots \sum_{j=1}^{N} \Xi \frac{h_j^3}{3!} \;\; \sum_{j=1}^{N} \Xi \frac{h_j^4}{4!} \cdots \sum_{j=1}^{N} \Xi \frac{h_j^2 k_j^2}{4} \right)^T \tag{46}$$

where

$$\Xi = \left[ -U_0 + U_j - \frac{1}{5!}\left( h_j \frac{\partial u_0}{\partial x} + k_j \frac{\partial u_0}{\partial y} \right)^{(5)} - \frac{1}{6!}\left( h_j \frac{\partial u_0}{\partial x} + k_j \frac{\partial u_0}{\partial y} \right)^{(6)} - \cdots \right] w^2(h_j, k_j) \tag{47}$$

with $N \geq 14$.

$b_{14}^*$ can be split into two parts as follows

$$b_{14}^* = b_{14} + b_{14}^{**} \tag{48}$$

Where b14 is as determined by Eq. (22), and the additional terms b 4 and correspond to the newly introduced higher order derivatives integrated into the Taylor series expansion to bring the functions from B14[u] to B 14[u].

After that, we may use the inverse matrix A1 14 to get a more accurate estimate of the partial derivatives.

$$D_{u_{14}} = A_{14}^{-1} b_{14} + A_{14}^{-1} b_{14}^{**}.$$

As can be shown in Eq. (49), the GFDM uses the approximation A1 14 b14 for the case of plates (see [9,8]), and the truncation errors for spatial derivatives are thus provided by

$$TE_{(x,y)_{14}} = A_{14}^{-1} b_{14}^{**}. \quad (50)$$

Thus

$$TE_{(x,y)} = -C_2^2 A^{-1} \times \left( \sum_{j=1}^{N} \Upsilon h_j \;\; \sum_{j=1}^{N} \Upsilon k_j \;\; \sum_{j=1}^{N} \Upsilon \frac{h_j^2}{2!} \cdots \sum_{j=1}^{N} \Upsilon \frac{h_j^3}{3!} \;\; \sum_{j=1}^{N} \Upsilon \frac{h_j^4}{4!} \cdots \sum_{j=1}^{N} \Upsilon \frac{h_j^2 k_j^2}{4} \right)^T \quad (51)$$

where

$$\Upsilon = -\left[ \frac{1}{5!} \left( h_j \frac{\partial u_0}{\partial x} + k_j \frac{\partial u_0}{\partial y} \right)^{(5)} + \frac{1}{6!} \left( h_j \frac{\partial u_0}{\partial x} + k_j \frac{\partial u_0}{\partial y} \right)^{(6)} + \cdots \right] w^2(h_j, k_j) \quad (52)$$

and operating

$$TE_{(x,y)} = C_2^2 \left[ \sum_{j=1}^{N} \Psi_{1,j} \frac{\partial^5 U}{\partial x^5} + \cdots + \Psi_{i,j} \frac{\partial^6 U}{\partial x^6} + \cdots \right] + \Theta(h_j, k_j) \quad (53)$$

Where (hj, kj) is a sequence of third- and higher-order functions, and (i,j) are homogeneous rational functions of order two. The inaccuracy introduced by truncating spatial derivatives in the case of plates is given by Eq. (53). Remember that the sum of the truncation errors (TTE) is

$$TTE = TE_t + TE_{(x,y)} \quad (54)$$

Where Eqs. (30) And (53) determine TEt and TE(x,y), respectively.

By analyzing Eq. 54 for derivatives with bounds, we can

$$\lim_{(\Delta t, h_j, k_j) \to (0,0,0)} TTE \to 0. \quad (55)$$

Then, Eq. (55) demonstrates the approximation's consistency under the truncation error condition.

## Criterion for stability

Stability of the difference schemes may be guaranteed by satisfying the von Neumann criterion, which is both sufficient and required [14]. The von Neumann technique theoretically only works for pure initial value issues with periodic beginning data, because it ignores boundary conditions. However, it does provide the essential requirements for stability of constant coefficient problems under any boundary conditions.

## Beams

The basic concept for the stability analysis is to do a harmonic decomposition of the estimated solution at grid points and at a certain time step n. After that, the approximate finite difference solution may be written in the star's nodes at time n

$$u_0^n = \xi^n e^{i v x_0}; \qquad u_j^n = \xi^n e^{i v x_j}$$

Where stands for the multiplier,.

$$x_j = x_0 + h_j; \qquad \xi = e^{-i w \Delta t}$$

Since the stability requirement may be expressed as 1, where the wave number, v, and 1 are is the threshold value. Simply by plugging Eq. (56) into Eq. (12), we see that the elimination of neix0 yields

$$\xi = 2 + \frac{1}{\xi} - (\Delta t)^2 C^1 \left( \eta_0 + \sum_{1}^{N} \eta_j e^{i v h_j} \right). \quad (57)$$

The quadratic equation may be calculated using Eq. (9) as an input.

$$\xi^2 - \xi \left[ 2 + C_1^2 (\Delta t)^2 \left( \sum_{1}^{N} \eta_j (1 - \cos v h_j) - i \sum_{1}^{N} \eta_j \sin v h_j \right) \right] + 1 = 0. \quad (58)$$

Therefore, the possible values are

$$\xi = b \pm \sqrt{b^2 - 1} \quad (59)$$

where

$$b = 1 + \frac{C_1^2 (\Delta t)^2}{2} \sum_{1}^{N} \eta_j (1 - \cos v h_j) - i \frac{C_1^2 (\Delta t)^2}{2} \sum_{1}^{N} \eta_j \sin v h_j. \quad (60)$$

Now, if we focus on the stability requirement, we get

$$\left\| b \pm \sqrt{b^2 - 1} \right\| \leq 1.$$

The star's stability condition is achieved by solving Eqs. (60) and (61), which include cancelling conservative conditions.

$$\Delta t \leq \frac{1}{4 C_1 \sqrt{|\eta_0|}} \quad (62)$$

Where C1 is the coefficient supplied by Equation (1) and 0 is the coefficient of the fourth-order estimate for the value of the function at the centre node of the star in Equation (9).

## Plates

The basic concept for the stability analysis is to do a harmonic decomposition of the estimated solution at grid points and at a certain time step n. Then, at time n, the finite difference approximation may be expressed in terms of star nodes.

$$u_0^n = \xi^n e^{i\mathbf{v}^T \mathbf{x}_0}; \quad u_j^n = \xi^n e^{i\mathbf{v}^T \mathbf{x}_j} \tag{63}$$

where $\xi$ is the amplification factor,

$$\mathbf{x}_j = \mathbf{x}_0 + \mathbf{h}_j; \quad \xi = e^{-iw\Delta t}.$$

This allows us to express the stability criterion as 1.

Cancellation of neivT x0 occurs when Eq. (63) is substituted into Eq. (26), producing

$$\xi = 2 + \frac{1}{\xi} - (\Delta t)^2 C_2^2 \left( \mu_0 + \sum_1^N \mu_j e^{i\mathbf{v}^T \mathbf{h}_j} \right). \tag{64}$$

The quadratic equation is found using Eq. (24) and some more mathematics.

$$\xi^2 - \xi \left[ 2 + C_2^2 (\Delta t)^2 \left( \sum_1^N \mu_j (1 - \cos \mathbf{v}^T \mathbf{h}_j) - i \sum_1^N \mu_j \sin \mathbf{v}^T \mathbf{h}_j \right) \right] + 1 = 0. \tag{65}$$

Therefore, the possible values are

$$\xi = b \pm \sqrt{b^2 - 1} \tag{66}$$

where

$$b = 1 + \frac{C_2^2 (\Delta t)^2}{2} \sum_1^N \mu_j (1 - \cos \mathbf{v}^T \mathbf{h}_j) - i \frac{C_2^2 (\Delta t)^2}{2} \sum_1^N \mu_j \sin \mathbf{v}^T \mathbf{h}_j. \tag{67}$$

Now, if we take the criterion for stability into account, we get

$$\left| b \pm \sqrt{b^2 - 1} \right| \le 1. \tag{68}$$

By cancelling out conservative conditions in Eqs. (67) And (68), we get the star stability condition, which reads as

$$\Delta t \le \frac{1}{4 C_2 \sqrt{|\mu_0|}} \tag{69}$$

Where C2 is the coefficient from Eq. (15) and 0 is the coefficient from Eq. (23) describing the fourth-order approximation of the value of the function at the central node of the star?

## Irregularity of the star (IIS) and stability

### Beams

In this part, we will define both the star's index of irregularity (IIS) and the node cloud's index of irregularity (IIC ). Coefficient 0 depends on (a) the total number of star nodes (including the central node), (b) the coordinates of each star node (including the central node), and (c) the weighting function (see Refs. [3,4,6]). Since Eq. (62) depends on the coordinates of the star's centre node, the number of star nodes and the weighting function are assumed to be constants. The average distance between a star's nodes and the centre node at coordinate (x0) is denoted by the symbol 0, while the average distance between stars in a cloud of nodes is denoted by the symbol.

$$\overline{\eta_0} = \eta_0 \tau^4. \tag{70}$$

It is possible to reformulate the stability requirement as

$$\Delta t < \frac{\tau^2}{4 C_1 \sqrt{|\overline{\eta_0}|}}. \tag{71}$$

Inequality (71) is written as follows for one-dimensional situations with regular mesh.

$$\Delta t < \frac{\sqrt{2}\tau^2}{18 C_1 \sqrt{3}} \quad \text{if } N = 4. \tag{72}$$

$$\frac{9\sqrt{3}}{2\sqrt{2|\overline{\eta_0}|}} \quad \text{if } N = 4 \tag{73}$$

The result is an unequal number (71).

For each star in the cloud of nodes, we provide the IIS for a star with the core node in (x0) as

$$IIS_{x_0} = \frac{9\sqrt{3}}{2\sqrt{2|\overline{\eta_0}|}} \quad \text{if } N = 4$$

That equals 1 if the mesh is regular and 0 if the IIS is not.

If IIS goes down, t goes down since Eq. (71) says it must, as a rise in the absolute value of t0 also means a reduction in t. When all the irregularity indices of the stars in a cloud of nodes are added together, the result is the irregularity index of the cloud of nodes (IIC).

$$\overline{\mu_0} = \mu_0 \tau^4.$$

$$\Delta t < \frac{r^2}{4C_2\sqrt{|\mu_0|}}. \quad (76)$$

For the regular mesh case, inequality (76) is for the cases of two dimensions as follows

$$\Delta t < \frac{9r^2}{C_2\sqrt{13}[3(1+\sqrt{2})+2\sqrt{5}]^2} \quad \text{if } N = 24. \quad (77)$$

Multiplying the right-hand side of inequality (77) by the factor

$$\frac{\sqrt{13}[3(1+\sqrt{2})+2\sqrt{5}]^2}{36\sqrt{|\mu_0|}} \quad \text{if } N = 24 \quad (78)$$

One obtains inequality (76).

We characterize the IIS for a star with a centre node in coordinates (x0, y0) as

$$IIS_{(x_0,y_0)} = \frac{\sqrt{13}[3(1+\sqrt{2})+2\sqrt{5}]^2}{36\sqrt{|\mu_0|}} \quad \text{if } N = 24 \quad (79)$$

## Beam analysis using numerical methods

This section employs a weighting function where

$$\Omega(h_j) = \frac{1}{\left(\sqrt{h_j^2}\right)^3}. \quad (80)$$

The worldwide precise inaccuracy may be determined via the formula

$$\text{Global exact error} = \sqrt{\frac{\sum_{i=1}^{NT} e_i^2}{NT}} \quad (81)$$

### Transverse vibrations of a simply supported beam

Let us solve the pde

$$\frac{\partial^2 U(x,t)}{\partial t^2} + \frac{1}{\pi^4}\frac{\partial^4 U(x,t)}{\partial x^4} = 0 \quad x \in (0,1), t > 0 \quad (82)$$

with boundary conditions

$$\begin{cases} U(0,t) = U(1,t) = 0 \\ \left.\frac{\partial^2 U(x,t)}{\partial x^2}\right|_{(0,t)} = \left.\frac{\partial^2 U(x,t)}{\partial x^2}\right|_{(1,t)} = 0, \end{cases} \quad (83)$$

and initial conditions

$$U(x,0) = 0; \quad \left.\frac{\partial U(x,t)}{\partial t}\right|_{(x,0)} = \sin(\pi x). \quad (84)$$

The exact solution is

$$U(x,t) = \sin(\pi x)\sin t. \quad (85)$$

### Forced vibrations of a simply supported beam

Here, we use a weighting function of (80), and the worldwide error is computed as (81).

For the pde, we have

$$\frac{\partial^2 U(x,t)}{\partial t^2} + \frac{1}{\pi^4}\frac{\partial^4 U(x,t)}{\partial x^4} = 15\sin(2\pi x)\sin t \quad x \in (0,1), t > 0 \quad (86)$$

with boundary conditions

$$\begin{cases} U(0,t) = U(1,t) = 0 \\ \left.\frac{\partial^2 U(x,t)}{\partial x^2}\right|_{(0,t)} = \left.\frac{\partial^2 U(x,t)}{\partial x^2}\right|_{(1,t)} = 0, \end{cases} \quad (87)$$

and initial conditions

$$U(x,0) = 0; \quad \left.\frac{\partial U(x,t)}{\partial t}\right|_{(x,0)} = \sin(\pi x) + \sin(2\pi x). \quad (88)$$

The exact solution for this case is given by

$$U(x,t) = (\sin(\pi x) + \sin(2\pi x))\sin t. \quad (89)$$

$$\frac{\partial^2 U(x,t)}{\partial t^2} + \frac{1}{\pi^4}\frac{\partial^4 U(x,t)}{\partial x^4} = 15\sin(2\pi x)\sin t \quad x \in (0,1), t > 0$$

### Transverse vibrations of a beam with one end fixed and other end free

Here, we use a weighting function of (80), from which we get the following formula for the worldwide exact error: (81). For those interested in the pde:

$$\frac{\partial^2 U(x,t)}{\partial t^2} + \frac{1}{1.875^4}\frac{\partial^4 U(x,t)}{\partial x^4} = 0 \quad x \in (0,1), t > 0 \quad (90)$$

with boundary conditions

$$\begin{cases} U(0,t) == 0 \\ \left.\frac{\partial U(x,t)}{\partial x}\right|_{(0,t)} = \left.\frac{\partial^2 U(x,t)}{\partial x^2}\right|_{(1,t)} = \left.\frac{\partial^3 U(x,t)}{\partial x^3}\right|_{(1,t)} = 0, \end{cases} \quad (91)$$

and initial conditions

$$U(x,0) = 0; \quad \left.\frac{\partial U(x,t)}{\partial t}\right|_{(x,0)} = \cos(1.875x) - \cosh(1.875x) - 0.7340327[\sin(1.875x) - \sinh(1.875x)]. \quad (92)$$

The exact solution is given by

$$U(x,t) = (\cos(1.875x) - \cosh(1.875x) - 0.7340327[\sin(1.875x) - \sinh(1.875x)])\sin t. \quad (93)$$

**Table 1**
Summary of several cases of beams.

| PDE | Boundary conditions | Initial conditions | Exact solution |
|---|---|---|---|
| Eq. (82) | Eq. (83) | Eq. (84) | Eq. (85) |
| Eq. (86) | Eq. (87) | Eq. (88) | Eq. (89) |
| Eq. (90) | Eq. (91) | Eq. (92) | Eq. (93) |
| Eq. (94) | Eq. (95) | Eq. (96) | Eq. (97) |

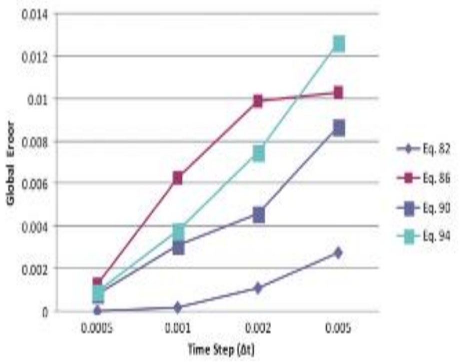



Fig. 3. Global error versus time step

**Transverse vibrations of a beam with one end fixed and other end simply supported**

Here, we use a weighting function of (80), from which we get the following formula for the worldwide exact error: (81). For those interested in the pde:

$$\frac{\partial^2 U(x,t)}{\partial t^2} + \frac{1}{3.927^4}\frac{\partial^4 U(x,t)}{\partial x^4} = 0 \quad x \in (0,1), t > 0 \qquad (94)$$

with boundary conditions

$$\begin{cases} U(0,t) = U(1,t) = 0 \\ \left.\frac{\partial U(x,t)}{\partial x}\right|_{(0,t)} = \left.\frac{\partial^2 U(x,t)}{\partial x^2}\right|_{(1,t)} = 0, \end{cases} \qquad (95)$$

and initial conditions

$$U(x,0) = 0$$

$$\left.\frac{\partial U(x,t)}{\partial t}\right|_{(x,0)} = 3.240366(\cos(7.069x) - \cosh(7.069x) - 1.000002[\sin(7.069x) - \sinh(7.069x)]). \qquad (96)$$

The exact solution is given by

$$U(x,t) = (\cos(7.069x) - \cosh(7.069x) - 1.000002[\sin(7.069x) - \sinh(7.069x)]) \sin 3.240366t. \qquad (97)$$

**Summary of the results obtained for beams**

We summaries the partial differential equations (PDEs), boundary conditions (BCs), starting conditions (ICs), and exact solutions (XSs) in Table 1. You can see the point clouds used in each of the four scenarios in Figures 1 and 2.

In all circumstances when the time step is less than the stability limit, as illustrated in Fig. 3, the global error rises with the time step (62). Fig. 4 displays the result of a drop in the cloud's index of irregularity, which leads to a little rise in the resulting inaccuracy (the IIC is in interval [0, 1], and it is equal to 1 when the cloud corresponds to the regular case).

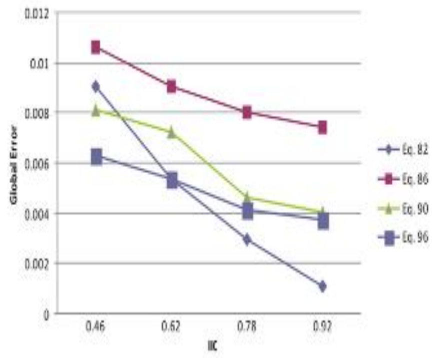



Fig. 4. Global error versus IIC

**Numerical results of plates**

This section employs a weighting function where

$$\Omega(h_j, k_j) = \frac{1}{\left(\sqrt{h_j^2 + k_j^2}\right)^3} \qquad (98)$$

And the worldwide precise error is found by (81).

Here, we show off a variety of plate-related numerical findings for the following use-cases. One, the unrestricted movement of a flat plate on a single support. Impact-induced free vibrations in a plate with a single support point. Induced vibrations in a plate with little support. Free vibrations of a stationary plate. We provide the PDEs, boundary conditions, beginning conditions, and precise solutions for all four scenarios below.

**Free vibrations of a simply supported plate**

The pde is

$$\frac{\partial^2 U(x,y,t)}{\partial t^2} + \frac{1}{4\pi^4}\left[\frac{\partial^4 U(x,y,t)}{\partial x^4} + 2\frac{\partial^4 U(x,y,t)}{\partial x^2 \partial y^2} + \frac{\partial^4 U(x,y,t)}{\partial y^4}\right]$$
$$= 15 \sin t \sin(2\pi x)\sin(2\pi y) \quad (x,y) \in (0,1) \times (0,1), t > 0 \qquad (99)$$

with boundary conditions

$$\begin{cases} U(x,y,t)|_\Gamma = 0 \\ \left.\frac{\partial^2 U(x,y,t)}{\partial y^2}\right|_{(0,y,t)} = \left.\frac{\partial^2 U(x,y,t)}{\partial y^2}\right|_{(1,y,t)} = 0, \quad \forall y \in [0,1] \\ \left.\frac{\partial^2 U(x,y,t)}{\partial x^2}\right|_{(x,0,t)} = \left.\frac{\partial^2 U(x,y,t)}{\partial x^2}\right|_{(x,1,t)} = 0, \quad \forall x \in [0,1] \end{cases} \qquad (100)$$

The starting condition, the domain border, and the interval [0, 1] [0, 1].

$$U(x,y,0) = 0; \quad \left.\frac{\partial U(x,y,t)}{\partial t}\right|_{(x,y,0)} = \sin(\pi x)\sin(\pi y). \qquad (101)$$

The exact solution is given by

$$U(x,y,t) = \sin(\pi x)\sin(\pi y)\sin t. \qquad (102)$$

**Free vibrations of a simply supported plate due to impact given to a point**

$$U(x,y,0)=0;\quad \begin{cases} \left.\dfrac{\partial U(x,y,t)}{\partial t}\right|_{(x,y,0)} = 1 & \text{if } x=y=0.5 \\ \left.\dfrac{\partial U(x,y,t)}{\partial t}\right|_{(x,y,0)} = 0 & \text{if } (x,y)\neq(0.5,0.5). \end{cases} \tag{103}$$

The exact solution is given by

$$U(x,y,t) = 2\left[\sin(\pi x)\sin(\pi y)\sin(t) - \frac{1}{9}\sin(3\pi x)\sin(3\pi y)\sin(9t) + \frac{1}{25}\sin(5\pi x)\sin(5\pi y)\sin(25t) - \cdots\right]. \tag{104}$$

### Forced vibrations of a simply supported plate

Here, we use weighting function (98), and we compute the worldwide error using (81)

For those interested in the pde:

$$\frac{\partial^2 U(x,y,t)}{\partial t^2} + \frac{1}{4\pi^4}\left[\frac{\partial^4 U(x,y,t)}{\partial x^4} + 2\frac{\partial^4 U(x,y,t)}{\partial x^2 \partial y^2} + \frac{\partial^4 U(x,y,t)}{\partial y^4}\right] = 15\sin t\sin(2\pi x)\sin(2\pi y) \quad (x,y)\in(0,1)\times(0,1), t>0 \tag{105}$$

Having Boundaries

$$\begin{cases} U(x,y,t)|_\Gamma = 0 \\ \left.\dfrac{\partial^2 U(x,y,t)}{\partial y^2}\right|_{(0,y,t)} = \left.\dfrac{\partial^2 U(x,y,t)}{\partial y^2}\right|_{(1,y,t)} = 0, & \forall y\in[0,1] \\ \left.\dfrac{\partial^2 U(x,y,t)}{\partial x^2}\right|_{(x,0,t)} = \left.\dfrac{\partial^2 U(x,y,t)}{\partial x^2}\right|_{(x,1,t)} = 0, & \forall x\in[0,1] \end{cases} \tag{106}$$

and initial conditions

$$U(x,y,0)=0;\quad \left.\frac{\partial U(x,y,t)}{\partial t}\right|_{(x,y,0)} = \sin(\pi x)\sin(\pi y) + \sin(2\pi x)\sin(2\pi y). \tag{107}$$

The exact solution is given by

$$U(x,y,t) = (\sin(\pi x)\sin(\pi y) + \sin(2\pi x)\sin(2\pi y))\sin t. \tag{108}$$

### Free vibrations of a fixed plate

$$\frac{\partial^2 U(x,y,t)}{\partial t^2} + \frac{1}{4(4.73)^4}\left[\frac{\partial^4 U(x,y,t)}{\partial x^4} + 2\frac{\partial^4 U(x,y,t)}{\partial x^2 \partial y^2} + \frac{\partial^4 U(x,y,t)}{\partial y^4}\right] = 15\sin t\sin(2\pi x)\sin(2\pi y) \quad (x,y)\in(0,1)\times(0,1), t>0 \tag{109}$$

### Having Boundaries

$$\begin{cases} U(x,y,t)|_\Gamma = 0 \\ \left.\dfrac{\partial U(x,y,t)}{\partial y}\right|_{(0,y,t)} = \left.\dfrac{\partial U(x,y,t)}{\partial y}\right|_{(1,y,t)} = 0, & \forall y\in[0,1] \\ \left.\dfrac{\partial U(x,y,t)}{\partial x}\right|_{(x,0,t)} = \left.\dfrac{\partial U(x,y,t)}{\partial x}\right|_{(x,1,t)} = 0, & \forall x\in[0,1] \end{cases} \tag{110}$$

If the starting conditions are [0, 1] [0, 1] and is the border of the domain

$$\begin{cases} U(x,y,0)=0 \\ \left.\dfrac{\partial U(x,y,t)}{\partial t}\right|_{(x,y,0)} = (\cos(4.73x) - \cosh(4.73x) - 0.982501[\sin(4.73x) \\ \quad - \sinh(4.73x)])(\cos(4.73y) - \cosh(4.73y) - 0.982501[\sin(4.73y) - \sinh(4.73y)]). \end{cases} \tag{111}$$

The exact solution is given by

$$U(x,y,t) = (\cos(4.73x) - \cosh(4.73x) - 0.982501[\sin(4.73x) - \sinh(4.73x)])(\cos(4.73y) - \cosh(4.73y) - 0.982501[\sin(4.73y) - \sinh(4.73y)])\sin t. \tag{112}$$

### Summary of the results obtained for plates

The PDEs, boundary conditions, beginning conditions, and precise solutions are summarized in Table 2. In Figs. 5 and 6, we see the point clouds used in these four scenarios.

**Table 2**
Summary of several cases of plates.

| PDE | Boundary conditions | Initial conditions | Exact solution |
|---|---|---|---|
| Eq. (99) | Eq. (100) | Eq. (101) | Eq. (102) |
| Eq. (99) | Eq. (100) | Eq. (103) | Eq. (104) |
| Eq. (105) | Eq. (106) | Eq. (107) | Eq. (108) |
| Eq. (109) | Eq. (110) | Eq. (111) | Eq. (112) |

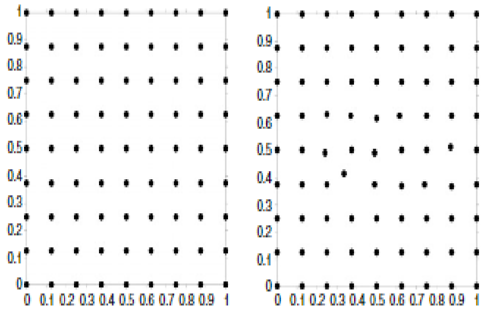
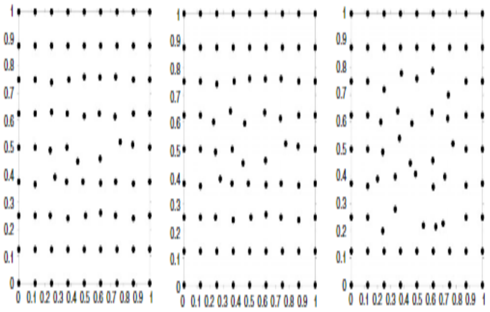


Fig. 5. Regular mesh (IIC = 1). Irregular mesh (IIC = 0.92).



Fig;6 Irregular mesh (IIC = 0.83). Irregular mesh (IIC = 0.76). Irregular mesh (IIC = 0.58

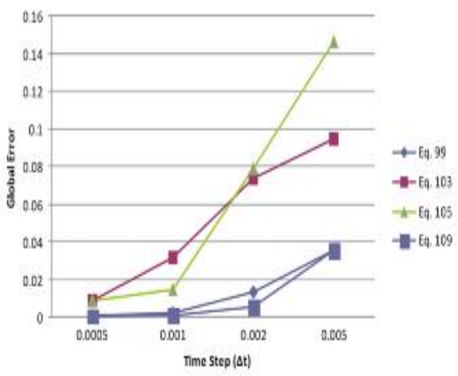



**Fig. 7.** Global error versus time step.

In all circumstances when the time step is below the stability limit, as illustrated in Fig. 7, the global error grows in proportion to the time step (69). Fig. 8 shows a little rise in inaccuracy when the cloud's index of irregularity is decreased (the IIC lies in the range [0, 1], and it is equal to 1 when the cloud matches the regular case).

## Convergence test

As can be shown in Fig. 9, the GFD approach converges reliably when applied to plates. When applied to case 6.1, Fig. 5 (cases with 121, 289, 441 and 676) for t = 0.005, the global error reduces as the number of nodes in the cloud of nodes rises.

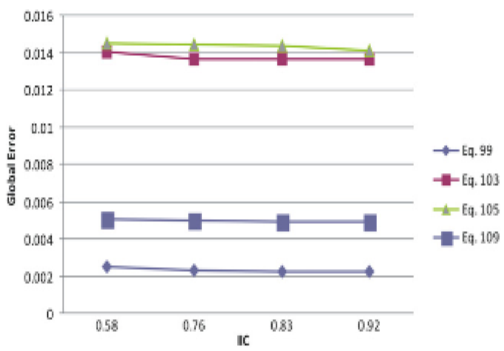



**Fig. 8.** Global error versus *IIC*.

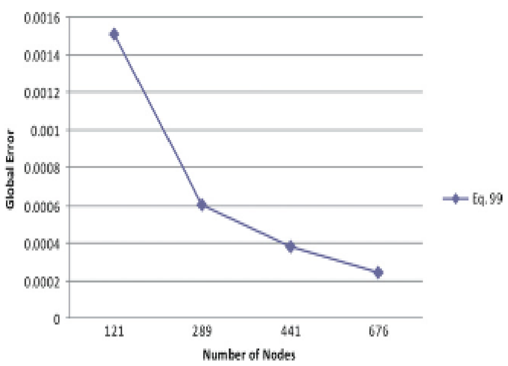



Fig. 9. Global error versus the number of nodes for case 6.1

## Conclusions

An intriguing approach to solving partial differential equations is the extended finite difference technique using uneven clouds of points. The generalized finite difference has been created to allow for the explicit solution of various issues in beam and plate dynamic analysis. For both beams and plates, the von Neumann stability criteria have been written as a function of the coefficients of the star equation for an irregular cloud of nodes. In this article, we provide an explanation of the index of irregularity of clouds of nodes (IIC) and how it relates to the stability bound. Numerical studies reveal that, when the time step is decreased (while being within stability bounds), the overall error decreases.

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