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A REVIEW ON HAMILTONIAN COLORINGS WITH MULTIPLE OBSERVATIONS

ABSTRACT

For vertices u and v in a connected graph G of order n, the length of a longest u–v path in G is denoted by D(u, v). A hamiltonian coloring c of G is an assignment c of colors (positive integers) to the vertices of G suchthat D(u, v) + |c(u) − c(v)|n − 1 for every two distinct vertices u and v of G. The value hc(c) of a hamiltonian coloring c of G is the maximum colorassigned to a vertex of G. The hamiltonian chromatic number hc(G) of Gis min{hc(c)} over all hamiltonian colorings c of G. Hamiltonian chromatic numbers of some special classes of graphs are determined. It is shown that for every two integers k and n with k1 an d n3, there exists a hamiltonian graph of order n with hamiltonian chromatic number k if andonly if 1kn − 2. Also, a sharp upper bound for the hamiltonian chromatic number of a connected graph in terms of its order is established.

1. Introduction

For a connected graph G of order n and diameter d and an integer k with 1kd, a radio k-coloring of G is defined in [1] as an assignment c of colors(positive integers) to the vertices of G such that

d(u, v) + |c(u) − c(v)|1 + k

for every two distinct vertices u and v of G. The value rck(c) of a radio k-coloring c of G is the maximum color assigned to a vertex of G; while the radio k-chromatic number rck(G) of G is min{rck(c)} over all radio k-colorings c of G. A radio k-coloring c of G is a minimum radio k- coloring if rck(c) = rck(G). These concepts were inspired by the so-called channel assignment problem, where channels are assigned to FM radio stations according to the distances between the stations (and some other factors as well)

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Since rc1(G) is the chromatic number (G), radio k-colorings provide a generalization of ordinary colorings of graphs. The radio d-chromatic number was studied in [1,2] and was also called the radio number. Radio d-colorings are also referred to as radio labelings since no two vertices can be colored the same in a radio d-coloring. Thus, in a radio labeling of a connected graph of diameter d, the labels (colors) assigned to adjacent vertices must differ by at least d, the labels assigned to two vertices whose distance is 2 must differ by at least d − 1, and so on, up to the vertices whose distance is d, that is, antipodal vertices, whose labels are only required to be different. A radio (d − 1)-coloring is less restrictive in that colors assigned to two vertices whose distance is i, where 1id, are only required to differ by at least d − i. In particular, antipodal vertices canbe colored the same. For this reason, radio (d − 1)-colorings are also called radio antipodal colorings or, more simply, antipodal colorings. Antipodal colorings of graphs were studied in [3,4], where rcd−1(G) was written asac(G).

Two ofthe major areas in graph theory are colorings and the study oflongest paths and cycles. Within the second area is hamiltonian graph theory, which includes a number of theorems that give sufficient conditions for graphs to contain hamiltonian cycles or cycles ofsome prescribed length. Another major topic ofstudy in hamiltonian graph theory is hamiltonian-connected graphs (graphs containing a hamiltonian u–v path for every pair u, v ofdistinct vertices). It is the goal ofthis paper to study a connection between these two areas.

Radio k-coloring of paths were studied in [5] for all possible valuesof k. In the case of an antipodal coloring of the path Pn of order n3 (and diameter n − 1), only the end-vertices of Pn are permitted to be colored the same since the only pair of antipodal vertices in Pn are its two end-vertices. Of course, the two end-vertices of Pn are connected by a hamiltonian path. As mentioned earlier, if u and v are any two distinct vertices of Pn and d(u, v) = i, then |c(u) − c(v)|n − 1 − i. Since Pn is a tree, not only is i the length of a shortest u–v path in Pn, it is, in fact, the length of any u–v path in Pn since every two vertices are connected by a unique path. In particular, the length of a longest u–v path in Pn is i as well. For vertices u and v in a connected graph G, let D(u, v) denote the length of a longest u–v path in G. Thus for every connected graph G of order n and diameter d, both d(u, v) and D(u, v) are metrics on V (G). Radio k-colorings of G are

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inspired by radio antipodal colorings c which are defined by the inequality

$$d(u, v) + |c(u) - c(v)| d.$$

If G is a path, then(1) is equivalent to

$$D(u, v) + |c(u) - c(v)| n - 1,$$

which suggests an extension of the coloring c that satisfies (2) for an arbitrary connected graph G. A hamiltonian coloring c of G is an assignment of colors (positive integers) to the vertices of G such that D(u, v) + |c(u) − c(v)|n − 1 for every two distinct vertices u and v of G. Ina hamiltonian coloring of G, two vertices u and v canbe assigned the samecolor only if G contains a hamiltonian u–v path. The value hc(c) of a hamiltonian coloring c of G is the maximum color assigned to a vertex of G. The hamiltonian chromatic number hc(G) of G is min{hc(c)} over all hamiltonian colorings c of G. A hamiltonian coloring c of G is a minimum hamiltonian coloring if hc(c) = hc(G).

A graph G is hamiltonian-connected (1) if for every pair u, v of distinct vertices of G, there is a hamiltonian u–v path. Consequently, we have thefollowing fact.

(2)

OBSERVATION

Let G be a connected graph. Then hc(G) = 1 if and onlyif G is hamiltonian-connected.

In a certain sense, the hamiltonian chromatic number of a connected graph G measures how close G is to being hamiltonian-connected, the nearer the hamiltonian chromatic number of a connected graph G is to 1, the closer G is to being hamiltonian-connected

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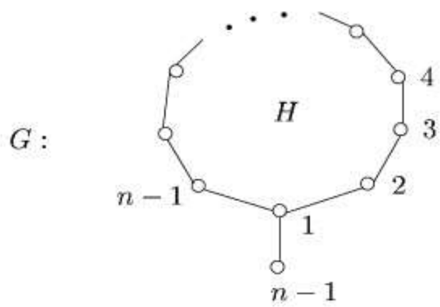

Fig. 1. A hamiltonian coloring c_0 of G.

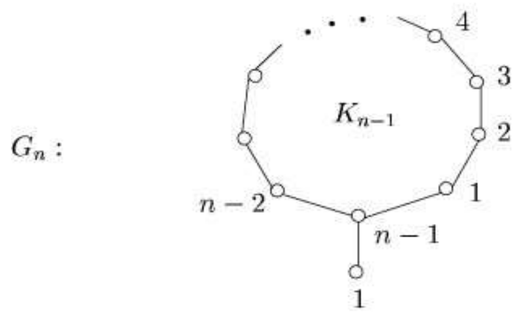

Fig. 2. An antipodal coloring c' of G_n.

2. Graphs with equal hamiltonian chromatic number and antipodal chromatic number

Since the path Pn is the only graph G of order n for which diam G = n − 1, we have the following.

Observation

If G is a path, then hc(G) = ac(G)

In [4] it was shown that ac(Pn) n−1 2 + 1 for every positive integer n. Moreover, it was shown in [5] that ac(Pn) n−1 2 − (n − 1)/2 + 4 for odd integers n7. Therefore, we have the following.

Lemma 2.2. Let H be a hamiltonian graph of order n − 13. If G is a graph obtained from H by adding a pendant edge, then hc(G) = n − 1.

Proof. Let C : v1, v2,...,vn−1, v1 be a hamiltonian cycle of H and let v1vn be the pendant edge of G. Let c be a hamiltonian coloring of G. Since D(u,v)n−2 for all u, v ∈ V (C), there is no pair of vertices in C that are coloredthe same by c. This implies that hc(c)n−1 an d so hc(G)n − 1.

Define a coloring c0 of G by c0(vi) = i for 1in − 1 and c0(vn) = n −1 (see Fig. 1). We show that c0 is a hamiltonian coloring of G

For n4, let Gn be the graph obtained from the complete graph Kn−1 by adding a pendant edge. Then Gn has order n and diameter 2. Let V (Gn) = {v1, v2,...,vn}, where deg vn = 1 and vn−1vn ∈ E(G). By

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Lemma 2.3, hc(Gn) = n − 1. We now show that ac(Gn) = hc(Gn) = n − 1. Let c be an antipodal coloring of Gn. Since diam Gn = 2, it follows that the colorsc(v1), c(v2), . . . , c(vn−1) are distinct and so ac(Gn)n − 1. Moreover, the coloring c of Gn defined by c (vi) = i for 1in − 1, c (vn) = 1 is an antipodal coloring of Gn (see Fig. 2) and so ac(Gn)=n−1. Hence there is an infinite class of graphs G of diameter 2 such that hc(G) = ac(G).

We now show that there exists an infinite class of graphs G of diameter 3 such that hc(G) = ac(G).

3. Hamiltonian chromatic numbers of some special classes of graphs

Since the complete graph Kn is hamiltonian-connected, hc(Kn) = 1. We state this below for later reference.

Observation

For n1, hc(Kn) = 1.

We now consider the complete bipartite graphs Kr,s, beginning with Kr,r. The graph Kr,r has order n = 2r and is hamiltonian but is not hamiltonian- connected. For distinct vertices u and v of Kr,r,

Therefore, for a hamiltonian coloring of Kr,r, every two nonadjacent vertices must be colored differently (while adjacent vertices can be colored the same). This implies that hc(Kr,r) = (Kr,r) = r.

We now determine hc(Kr,s) with r less than s, beginning with r = 1.

If T is a spanning tree of a connected graph G, then hc(G)<hc(T). The following lemma will also be useful to us. The complement G of a graph G is the graph with vertex set V (G) such that two vertices are adjacent in G if and only if they are not adjacent in G.

If T is a tree of order at least 4 that is not a star, then T contains a hamiltonian path.
We proceed by induction on the order n of T. For n=4, the path P4 of order 4 is the only tree of order 4 that is not a star. Since P4 = P4, the result holds for n = 4. Assume that for every tree of order k−1>4 that is not a star, its complement contains a hamiltonian path. Now let T be a tree of order k that is not a star. Then T contains an end-vertex v such that T − v is not a star. By the induction hypothesis, T − v contains a hamiltonian path, say v1, v2,...,vk−1. Since v is anend-

$$D(u,v)=\begin{cases} n-1 & \text{if } uv\in E(K_{r,r}),\\ n-2 & \text{if } uv\notin E(K_{r,r}).\end{cases}$$

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vertex of T, it follows that v is adjacent to at most one of v1 and vk−1. Without loss of generality, assume that v1 and v are not adjacent in T. Then v and v1 are adjacent in T and so v, v1, v2,...,vk−1 is a hamiltonian path in T .

Let G be a connected graph of order n>5. If there exists a hamiltonian coloring c of G with hc>4 satisfying one of the following conditions:

(1) Seq(c) = (1, 2, hc(c) − 1, hc(c));

(2) Seq(c) = (1, hc(c) − 1, hc(c)) and there exists a c-pair S with c(S) = 1;

(3) Seq(c)=(1, 2, hc(c)) and there exists a c-pair S with c(S)=hc(c); then cir(G)>n−1.

CONCLUSION

It was shown that if T is a spanning tree of a connected graph G, then hc(G)<hc(T). It is clear that if G is a connected graph of order at least 4 that is not a star, then G is spanned by a tree that is not a star. Thus the following corollary is an immediate consequence of Theorem . There exists no connected graph G of order n5 such that $hc(G)=(n-2)^2$. Furthermore, if G is a connected graph of order n5 that is not a star, then $hc(G) (n-2)^2 - 1$. It was shown in [7] that $hc(G)(n-2)^2 + 1$ for every connected graph of order n>2. The identity holds ifand only if G is a star. It was also shown that if n = 5, then there exists no connected graph oforder n with hc(G) = (n − 2) 2. Corollary 4.3 is an extension ofthis result for all n>5.

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