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Low-Complexity MU-MIMO Nonlinear Pre coding Using Degree-2 Sparse Vector Perturbation

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Abstract—Multiuser multiple-input multiple-output (MU- MIMO) nonlinear precoding techniques face the problem of poor computational scalability to the size of the network. In this paper, the fundamental problem of MU-MIMO scalability is tackled through a novel signal-processing approach, which is called degree-2 vector perturbation (D2VP). Unlike the conventional VP approaches that aim at minimizing the transmit-to-receive energy ratio through searching over an N -dimensional Euclidean space, D2VP shares the same target through an iterative-optimization procedure. Each iteration performs vector perturbation over two optimally selected subspaces. By this means, the computational complexity is managed to be in the cubic order of the size of MU- MIMO, and this mainly comes from the inverse of the channel matrix. In terms of the performance, it is shown that D2VP offers comparable bit-error-rate to the sphere encoding approach for the case of small MU-MIMO. For the case of medium and large MU-MIMO when the sphere encoding does not apply due to unimplementable complexity, D2VP outperforms the lattice- reduction VP by around 5-10 dB in Eb/No and 10-50 dB in normalized computational complexity.

Index Terms—Low complexity, multiuser multiple-input multiple-output (MU-MIMO), nonlinear precoding, vector per-turbat

INTRODUCTION

NEXT generation of broadband mobile internet (namely 5G) is expected to support several orders of magnitude in capacity compared with that in 4G and its evolutions. There are several ways of achieving such huge capacity through densification of cells, massive multiple-input multiple-output (MIMO) and new extended bandwidth and their combination [1]. This paper presents a novel multiuser (MU) MIMO nonlinear precoding (NLP) approach for the downlink of MU-MIMO networks, of which the computational complexity scales linearly with the size of the MIMO networks, and the performance can be better than the state-of-the-art by 10 dB or more in Eb/No. broadcast channel. Theoretically, the sum-rate capacity of sucha channel grows linearly with the number of spatial-domain DoF [2], [3], and it can be achieved through multi-antenna dirty-paper coding (DPC) [3], [4]. However, the practical implementation of multi-antenna DPC faces great challenges of computational scalability to the size of the MIMO network;

and today only small-scale MU-MIMO (up to 8×8) with linearprecoding is adopted in 4G standards.

In the last decade, a number of remarkable contributions have been reported in the scope of multi-antenna DPC, which include nested lattice [5], [6], trellis precoding [7], V-BLAST precoding [8] and vector perturbation (VP) [9]. It has been shown that the VP technique can achieve near-optimum perfor-mance at all signal-to-noise ratios (SNRs) [10]. Nevertheless, the VP technique also faces big problems of the computational scalability. Specifically, the optimum VP technique aims at solving an integer least-square (ILS) problem, which theoreti- cally requires exhaustive search over an infinite set of integers in the N -dimensional Euclidean space. Such an approach costs infinite number of arithmetic operations, and thus it is not possible to implement. The sphere encoding (SE) VP approach proposed in [10] successfully avoids the problem of infinite searching by

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conducting tree searching over a finite set of integers. However, the computational complexity of SE-VP still scales exponentially with the size of the network.

One of sustainable approaches that can dramatically reduce the computational complexity is the lattice reduction (LR), which has been widely investigated for both the uplink MIMO detection [11] and the downlink MIMO nonlinear precoding (NLP) [12]. It has been shown that the LR-VP approach is a sub-optimum NLP at the cost of computational complex- ity between $O(N^4)$ and $O(N^5)$. Such a large reduction in computational complexity is certainly impressive. However, the complexity is still too high for the LR-VP approach to be implemented using the current digital-signal-processor (DSP) technology. It is possible to further trade off the performance for lower complexity through for instance the V-BLAST approach [8], [10]. However, the V-BLAST approach still costs expensive computational complexity, which is in the order of $O(N^4)$. Moreover, the performance of V-BLAST is only 2-3 dB better than that of linear zero-forcing (ZF) precoding.

Certainly, one can find more MU-MIMO NLP approaches in the literature (e.g. [13], [14]). Most of existing approaches were looking for a good tradeoff between the performance and complexity. The question is: is it possible to find a precoding technique that can show excellence in both the performance and complexity? If the answer is “yes”, then such a technique can bring MU-MIMO NLP much closer to fruition.

A. Contribution

Motivated by the above question, a novel MU-MIMO NLP approach, named degree-2 vector perturbation (D2VP), is proposed in this paper. It will be shown that the D2VP approach outperforms the LR-VP approach by around $5 - 10$ dB in SNR (subject to the size of MU-MIMO networks), and it manages the computational complexity in the order of $O(N^3)$, which mainly comes from the inverse of the MIMO channel matrix. Therefore, the D2VP approach offers competitive computational complexity in comparison to the linear ZF approach (see [15], [16]) with more than 10 dB performance improvement in SNR.

The basic idea of D2VP comes from an important phe- nomenon: for the VP technique aiming to minimize the transmit-to-receive energy ratio, the majority of the con- tribution comes from a small portion of the subspaces in the N -dimensional Euclidean space[1]. This means that the perturbation vector is sparse in nature. With this interesting phenomenon in mind, the VP optimization process does not need to search the entire Euclidean space for the global optimum point. Instead, it can break down the optimization process into several iterations, with each performing local optimization based on two optimally selected subspaces. By this means, given a finite set of integers with the size K, the complexity paid for exhaustive search in the two selected subspaces is $O(K^2)$. In fact, the exhaustive search is not needed in the D2VP optimization process. In Section III, it will be shown that D2VP forms a simple convex optimization problem, of which the local optimum point can be found in a closed form. This immediately reduces the optimization complexity from the square order to linear. In addition to the new concept, other major contributions of this paper include:

- Determine the subspaces of interest in the iterative pro- cess of D2VP. To this end, an optimum D2VP approach is developed through exhaustive search over all possible combinations of the subspaces. This approach ensures the best combination of subspaces with the complexity of $O(N(N - 1))$. In order to reduce the computational complexity, a complexity-reduced (CR) D2VP approach
- Generally, the perturbation vector can be either complex or real. In Section V, it will be shown that the real version offers comparable performance to the complex version when the size of MU-MIMO is sufficiently large (e.g. $N \geq 64$). Therefore, the real version can be a better approach for the case of large MU-MIMO due to its relatively low computational complexity.
- In fully-loaded MU-MIMO systems, most of VP-based NLP approaches do not get their performance improved when the size of MU-MIMO increases. Our computer simulations show that it is not the case for the D2VP approach. When the size of MU-MIMO is small (e.g. $N \leq 8$), the optimum D2VP approach has its perfor- mance improved with the increase of the MU-MIMO size. It means that the optimum D2VP can enjoy the spatial- domain diversity gain in the case of small MU-MIMO.
- In addition to the D2VP approach, we have also experi- mentally examined an extended approach, which is called degree-3 vector perturbation (D3VP). Although D3VP largely increases the computational complexity, our com- puter simulations show that it outperforms D2VP by up to 2 dB in SNR for the case of small MU-MIMO. This result encourages us to investigate the best performance- complexity tradeoff of the sparse vector perturbation, which could be a piece of interesting future work.

The rest of this paper is organized as follows. Section II is the preliminary section, which includes the system model of MU-MIMO, basic assumptions, concept of vector perturba- tion, as well as the problem formulation. The basic concept of D2VP and the algorithm optimization are presented in Section III. The RC-D2VP approach is presented in Section IV. Section V provides the simulation results and performance evaluation. The conclusion is drawn in Section VI.

PRELIMINARY

A. Vector Perturbation and Optimization

The original work of VP is built upon a discrete-time equivalent baseband model, which describes the link-level ofa wireless system including an access point with

M transmit antennas and N individual users. Each user has one receive antenna (see [9]). The observed signal

is proposed by selecting the subspaces which minimizethe impact of the largest singular values of

Hence, the ILS problem in (7) does not lead to a maximum- likelihood solution. Moreover, the linear minimum mean- square error approach widely used for the MU-MIMO detec- tion is not applicable to (7). On the other hand, the principle

of sphere decoding can still be employed to solve (7) as long as ω is restricted to a finite set of integers (this is known as the SE-VP approach); and the lattice-reduction approach can also be employed to regularize the channel inverse matrix

$\mathrm{H}^{\dagger}$. Nevertheless, as we have already discussed in Section I, none of existing approaches is saturated in terms of the

performance-complexity tradeoff, and thus the D2VP approach is motivated.

the technique with full CSIT provides the upper bound of the data rate.

Regarding the dimension and regularity of H, we found the

discussion in [9] already quite comprehensive. Here, we stress that the number of spatial-domain DoF equals to the rank of H, which puts an upper limit onto the number of orthogonal data-streams. Without loss of generality, we therefore can assume $M \geq N$ and $N = \mathrm{Rank}(\mathrm{H})$. It is worthwhile to note that future wireless networks will be super dense in nature,

and very demanding to the highly spectral efficiency. To this end, the number of spatial-domain DoF should be as large as possible, and thus the spatial domain is very

at the n^{th} user is

the channel

likely to be fully loaded. In this case, the linear MU-MIMO precoding techniques are far away from the optimum [22]. Therefore, in the rest of the paper, we consider the case $M = N$ for its critical position in future super dense networks.

1) User equality in date rate: In the original VP problem, it is assumed that all the users (receivers) have the same data rate(modulation). We also recognize this as a practical assumption. For instance in UMTS or LTE-A networks, the data rate is of- ten region specific. The data regions are classified according to the large-scale path loss between the transmitter and receivers [23], [24]. In this case, users located in the same data region can be scheduled with the VP-based spatial-domain multiple- access (SDMA), and those located in different data regions can be scheduled on different time or frequency resources.

2) Synchronization issues: We recognize synchronization

as one of critical issues in the area of MU-MIMO processing. The issue of timing synchronization can be relatively easy to solve by employing the time-domain guard interval or cyclic prefix. However, the frequency synchronization is indeed a big concern for the practical implementation of MU-MIMO systems. Nevertheless, there have been already a lot of on- going research activities in the scopes of synchronization and waveform design (e.g. [25]), and thus in this paper we assume

the perfect case of synchronization so that our technical presentation can be focused on the immediate problem of interest.

II. Degree-2 Sparse Vector Perturbation

A. Concept and Rationale

Definition 1: The sparsity of perturbation vector refers to the phenomenon: every element of ω has a large probability to be zero after the vector perturbation. Equivalently, when the size of ω is large, most of the elements in ω are zero.

According to the simulation results in [14], the probability

for an element of ω to be zero is around 80% or more. This phenomenon has been confirmed through our computer simulations (see Section V).

Definition 2: D2VP is a low-complexity vector perturbationtechnique, which takes advantage of the sparsity of perturba- tion vector in the matrix-regularization procedure. Similar to the original VP technique, the objective of D2VP is also to

handle the ILS problem (7). Instead of manipulating all the elements of the perturbation vector ω, the idea of D2VP is to break down the matrix-regularization process into several iterations, each performing the matrix-regularization based on two appropriately selected elements of ω.

Let us take the i^{th} iteration as an example to elaborate the basic concept of D2VP. Note that, for each iteration, the terms $\bar{s}$ and ω^* will be updated, and thus in the following

expressions they are labelled with the index (i). Then, the objective function of the i^{th} iteration is

$$\omega^*(i) = \mathrm{argmin}\|\bar{s}(i-1) - \alpha \mathrm{H}^{\dagger}\omega\|^2, \quad (8)$$

the complexity paid for searching over the finite set is in the square order. Note that the optimization procedure of

(11) requires to visit all possible states of (n_1, n_2), which also costs square-order complexity. Then, the overall cost of computational complexity is in the order of four, which is certainly too expensive. In Section III, we will demonstrate an optimum D2VP approach, with which the optimum point of (11) can be found in a closed form.

Iterative process and convergence: In general, D2VP is a sub-optimum VP approach due to the reduced number of subspaces (or equivalently the reduced number of non-

zero elements in ω) involved in the optimization procedure. Nevertheless, the optimality of D2VP can be improved through the iterative process described by (8)-(10).

Consider the outcome of the i^{th} iteration, i.e., $\omega^*(i)$ and $\bar{s}(i)$. The vector $\omega^*(i)$ has two possible states: 1) $\omega^*(i) = 0$, or 2) $\omega^*(i) \neq 0$. For the state of $\omega^*(i) = 0$, (9) shows that $\bar{s}(i) = \bar{s}(i-1)$, with which more iterations would not further improve the performance, and thus the iterative process terminates. For the state of $\omega^*(i) \;/\!= 0$, the objective function (8) assures $\|\bar{s}(i)\| < \|\bar{s}(i-1)\|$, which means that the performance of D2VP has been further improved. Note

that each iteration aims at reaching a local optimum based on the previous outcome $\bar{s}(i-1)$, therefore the iterative process functions as the neighbourhood search [26], which will quickly

converge to a local optimum solution.
and δ^2 is upper bounded by

of ω_r. Then, the objective function (11) still holds, and the optimum D2VP approach presented in Section III-B can be straightforwardly employed to obtain the optimum solution in the case of complex ω.
The only difference between the cases with complex and real version of ω is: the complex version requires searching over $N(2N-1)$ possible states of (n_1, n_2), and this increases the computational complexity by around 3 folds. On the other hand, since ω_c doubles the length of the perturbation vector, the error term ε_l in (14)-(16) is largely reduced. Moreover, it is easy to understand that the real version is a subset of the complex version. Therefore, the complex version should outperform the real version in terms of the performance optimality, and this conclusion has been confirmed through the computer simulations (see Section V).

B. Analysis of Computational Complexity

proach requires the computational complexity in order of four. This is because the variables I and L are not a function of N; and for most of the cases, we have $L \leq 2$ and $I \leq 3$. Therefore, the computational complexity for the real version

COMPLEXITY-REDUCED APPROACH OF D2VP Section III-D has shown that, apart from the inverse of the channel matrix, the major complexity of the optimum D2VP approach comes from searching over all possible states of (n_1, n_2). However, due to the randomness of the channel matrix H, it is unlikely to identify the best state of (n_1, n_2) without employment of the exhaustive search. Therefore, the objective of this section is to propose a complex-reduced (CR) approach of D2VP, which can avoid the exhaustive search at the price of the performance.

Given the singular-value decomposition (SVD): H = $V^H \Sigma U$, the SVD of $H^\dagger$ is
Indeed, finding the optimum solution of (34) still costs the computational complexity in an exponential order. However, we are able to find a D2VP approach, which offers a sub- optimum solution to (34).

Denote $v_{1,n}$ to be the n^{th} element of v_1. Fig. 2 shows an example of $N = 4$, where the elements of v_1 are randomly distributed in a complex plane. Two of the elements (i.e., $v_{1,3}$ and $v_{1,4}$) are composed into a new complex, which is very close to the complex $v_1^T s$. In the procedure of complex composition, the two real integers ω_3 and ω_4 are utilized to scale the complex elements $v_{1,3}$ and $v_{1,4}$, respectively.

In terms of computing, there are many ways of selecting the two elements of v_1 (or correspondingly the two elements of ω). A straightforward way is to exhaustively visit all possible combinations of any two components of v_1. This of course leads to the best selection at the price of a square-order of the computational complexity, and thus the exhaustive search is not a favourable approach. Here, we propose a simple approach of selection.

The overall computational complexity of the optimum D2VP approach is easy to calculate. Take the real version of ω as an example. For each iteration, the optimum D2VP approach spends the complexity of $\frac{N(N-1)}{}$ for the exhaustive search over all possible states of (n_1, n_2). Moreover, for each state, we need to search all candidates of δ_{n_1}. Given the total number of I iterations as well as the maximum of L candidates of δ_{n_1}, the overall computational complexity is

Fig. 1. Showcase the ratio of λ_n/λ_1 with respect to the size of MU-MIMO($N = 4, 12, 64, 128$).

according to the i.i.d. complex Gaussian distribution, Σ can have very small singular values, but the probability for a singular value to be zero is negligibly small. Hence, we have $\Sigma^\dagger = \Sigma^{-1}$ hold in general. With this fact in mind, we apply (27) into (7) and obtain

$$H^\dagger = U^H \Sigma^\dagger V, \tag{27}$$

where U, V is unitary matrix, and Σ a diagonal matrix with the singular values of H on its diagonal. When H is generated

It is of our interest to study the ratio $\lambda_n/\lambda_1, \forall n,$ with various configurations of the size of MU-MIMO, which is illustrated in Fig. 1. It is observed that the ratio λ_n/λ_1 drops rapidly with the increase of n. For most of the singular values, the ratio λ_n/λ_1 is negligibly small (< 0.1), and only a small portion of the singular values dominate the objective function (31). herefore, we can form an approximate version of (31)

Denote $\angle(v^T s)$ to be the phase of $v^T s$. The proposed approach first computes

By this means, we will have at most N candidates of ω_2, with which *Theorem 1* can be employed to determine the best one for performing the D2VP processing. However, the computational complexity of the RC-D2VP approach increases to $O(NN)$.

III. COMPUTER SIMULATIONS AND PERFORMANCE EVALUATION

The primary objective of computer simulations is to exam- ine the link-level scalability of the D2VP nonlinear precoding technique with respect to the size of MU-MIMO networks. The key performance metrics are the bit-error-rate (BER) performance and computational complexity. The baseline for performance comparison includes three techniques, which are the SE-VP, LR-VP, as well as the linear ZF precoding. The SE-VP technique should offer the best performance for the small size of MU-MIMO (e.g. $N = 4$). However, the computational complexity of SE-VP increases exponentially with size of MU-MIMO (e.g. $N \geq 8$), we employ the LR-VP

, (35) 1

technique as the main baseline due to its well-recognized performance-complexity tradeoff. In terms of the performance, which includes the angles and complementary angles between $v_{1,n}$ and v^T s. Then, we find the smallest positive value and the largest negative value of ψ_n. If these two values are corresponding to two different elements of $v_{1,n}$, then we said that the desired elements have been found; or otherwiseassuming $\psi_1(>0)$ and $\psi_2(<0)$ to be the angles of interest with $|\psi_1| > |\psi_2|$, we suggest to replace ψ_1 with the second smallest positive value of ψ_n. By this means, we are able to find two elements of v_1 (or equivalently ω), which can formthe linear ZF precoding surely performs the worst. However, it features the lowest computational complexity, and thus serves as an excellent baseline to evaluate the complexity cost of the D2VP technique. It is worthwhile to note that the V-BLAST based NLP technique is not employed for the performance comparison. This is mainly because the LR-VP technique has been proved to be better than the V-BLAST technique in terms of the performance [12], and we omit the V-BLAST results for the sake of delivering a concise presentation.

In our computer simulations, the MU-MIMO system was configured exactly the same as that has been introduced in Section II. Each entry of the channel matrix H was independently generated according to the complex Gaussian distribution with normalised variance. We recognize the fact that the entries of H can be statistically correlated in practice, and the wireless communication channels are mostly frequency selective. However, we note that the channel correlation will only reduce the number of DoF in the spatial domain, and this is equivalent to the case of reducing the size of uncorrelated MIMO in our simulation model. Moreover, we assume that the frequency-selective channel can be nicely converted into a number of parallel flat sub-channels through employment of multi-carrier transmissions, and our interest is focused on one of the sub-channels. In fact, similar simulation setup has been widely adopted by most of previous works in this scope. hannel realizations. The SNR is defined by the average received bit-energy per antenna to noise ratio. It is worthwhile to note that our simulations are mainly for an uncoded source with 4-QAM modulation (16-QAM will also be examined mainly for the sake of elaboration). This is because the VP technique has already been well evaluated for the coded sources and optimized for various order of QAM modulations (see [10]). Since the D2VP technique does not change the basic structure of VP, and our simulations are used mainly for the evaluation of large MU-MIMO systems, uncoded source is considered to be a more cost-effective option for the computer simulations.

Specifically, our computer simulations are structured into the following four experiments.

Experiment 1: The objective of this experiment is to examine the performance of the optimum D2VP approach when the perturbation vector ω is complex. Fig. 3 illustrates the BER results as a function of Eb/No for the case of 4 × 4 MU-MIMO (i.e., N = 4). Generally, the SE-VP approach gives

respect to the size of MU-MIMO, and thus for the medium and large

the best BER performance; and there is a large SNR gap (10 dB difference in Eb/No) observed between the SE-VP and the linear ZF precoding. The LR-VP approach shows up to 2 dB difference in Eb/No in comparison with the SE-VP approach. The maximal gap appears at the medium SNRs (Eb/No= 8- 10 dB). Moreover, at the high SNR (Eb/No= 20dB), the LR- VP approach performs even better than the SE-VP approach. This result is however not surprising. The major reason is that the SE-VP approach was searching for the optimum solution within a finite set of integers, i.e., {−j, −1, 0, 1, j}. Such approximation is acceptable at low and medium SNRs;
however it results in considerable optimality loss at high SNRs.

Now, let us take a close look of the optimum D2VP approach. It is observed that the D2VP approach offers very close performance to the SE-VP at low and medium SNRs (Eb/No≤ 10 dB). Moreover, it outperforms the LR-VP approach by around 2 dB in Eb/No. However, when Eb/No is as high as 16 dB or above, D2VP performs worse than the LR-VP approach. This is because the D2VP approach only utilizes two subspaces per iteration for conducting the vector perturbation,

Experiment 3: The objective of this experiment is to examine the performance of CR-D2VP as well as the performance-complexity tradeoff of various VP approaches. The performance comparison between the CR-D2VP and optimum D2VP is provided in Fig. 8. Both approaches use the complex vector perturbation. For the case of N = 4, the CR-D2VP approach (with N = 2) shows relatively close performance to the optimum D2VP approach particularly for the low and medium SNR range (Eb/No<12 dB). For the case of N = 256, the CR-D2VP approach (with N = 2) shows about 6 dB SNR gap when comparing to the optimum D2VP approach. It is clear that the configuration of N = 2 is too approximate in terms of the performance optimality. Surely, ulation. It is probably also interesting to see how the D2VP approaches behaviour for higher-order modulations. Since the optimum configuration of VP with respect to the QAM modulation schemes has been well studied in [10], here we pick up 16-QAM as a showcase to demonstrate the BER performance as a function of N (Eb/No=12 dB). The simulation results are depicted in Fig. 10. It is observed that the D2VP approaches significantly outperform the LR-VP approach. Particularly, theoptimum D2VP can largely improve the BER performance for
the case of N ≥ 12. Fig. 11 shows the computational complexity of NLP techniques, which is normalized by the complexity of ZF precoding. It is shown that the D2VP approaches can significantly reduce the computational complexity of NLP. The complexity reduction is about 10 − 50 dB (subject to the size of MU-MIMO) when comparing with the LR-VP approach.

More interestingly, the D2VP approaches show almost similar complexity as the ZF precoding when the size of MU-MIMO becomes large (e.g. N ≥ 12). This is because, in the case of large MU-MIMO, the computational complexity of D2VP is dominated by the operation of channel matrix inverse.

Experiment 4: The objective of this experiment is to examine the convergence behaviour of the D2VP approaches. Fig. 12 shows the BER performance of the optimum D2VP (complex version) with respect to the number of iterations. For the case of N = 4, the performance does not get improved after just two iterations. The difference between the 1st iteration and the 2nd iteration is negligibly small. For the case of N = 64, the performance of D2VP does not get considerably improved after three iterations.

Fig. 13 demonstrates the convergence behaviour of both the CR-D2VP (N = 2) and optimum D2VP for various cases of N . The CR-D2VP approach shows the performance converged after one or two iterations. The optimum D2VP gets its performance considerably improved by employing three iterations. For the case of large MU-MIMO (e.g. N ≥ 128), more iterations can further improve the performance of the optimum D2VP approach, although the improvement is not

Hence, the lower bound in (44) has to be no larger than the upper bound in (45). By solving this inequality we will get the upper bound in (24), and Theorem 1 is therefore proved.

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as large as the first three iterations. When the size of MU-MIMO is relatively small (e.g. N ≤ 64), the performance improvement is not considerable after three iterations.

CONCLUSION

In this paper, we have presented a novel multi-antenna nonlinear precoding technique, which demonstrated excellent performance and complexity scalability to the size of MU-MIMO networks. By exploiting the sparse nature of the perturbation vector, the proposed technique tackles the integer least-square optimization problem through several iterations, with each performs degree-2 vector perturbation. By this means, the N-dimensional ILS optimization problem is effectively

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